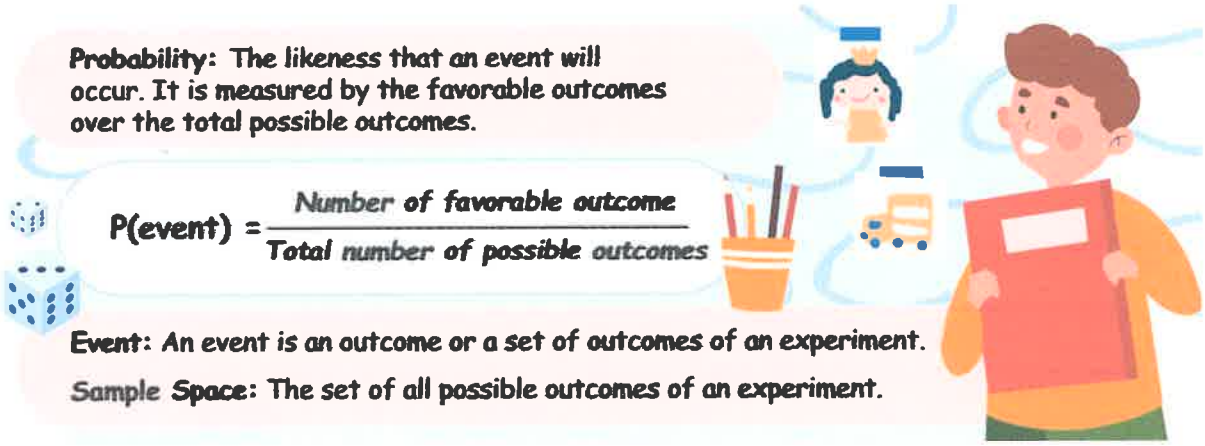
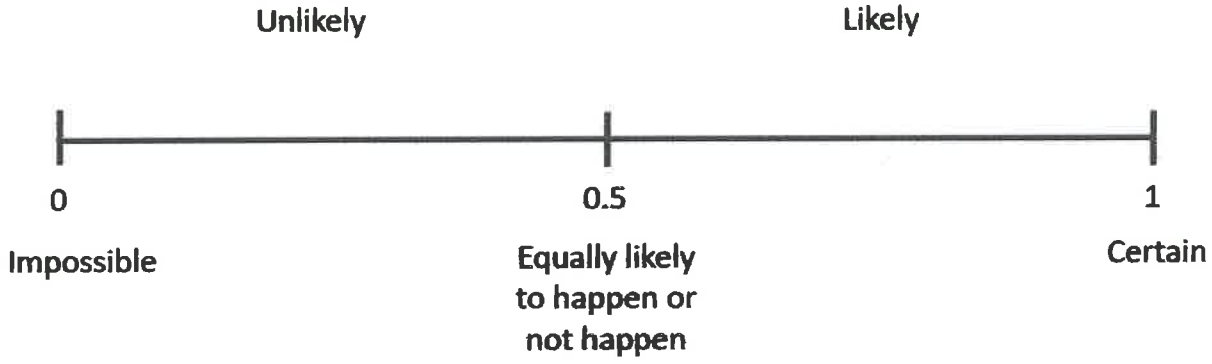
Chapter 3: Probability Topics

3.1: Terminology



- **Outcome** - the result of an experiment
- **Event** - any combination of outcomes
- **Sample Space** - the set of all possible outcomes of an experiment
- **Probability** - a long-term relative frequency of an outcome whose value is between zero and one, inclusive $\mathbf{0 \leq P(event) \leq 1}$
  - A probability of zero means that the event is **impossible** (0% chance of happening)
  - A probability of one means that the event is **certain to occur** (100% chance)



- **Equally Likely** - when each outcome of an experiment occurs with equal probability.
  - In a fair, six-sided die, it is equally likely to roll any of the numbers one through six.
    P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = $\frac{1}{6}$ $P(event) = \frac{number\ of\ correct\ choices}{total\ number\ of\ choices}$
  - In a fair coin, it is equally likely to get heads and tails.
    P(heads) = P(tails) = $\frac{1}{2}$

- The **complement** of event, *A*, denoted A', is the set of outcomes **not** in *A*.
  $P(A) + P(A') = 1$ or $P(A') = 1 - P(A)$

- The **conditional probability** of *A* given *B*, denoted *P*(*A*|*B*), is the probability that event *A* will occur given that event *B* has already occurred.

3.2: Independent and Mutually Exclusive Events

- Two events are **independent** if the knowledge that one occurred does not affect the chance the other occurs. Mathematically, this means that *P*(*A*|*B*) = *P*(*A*).

- Sampling **with replacement** means that members of the population are replaced after being selected and that each member can be chosen more than once. (known as **independent** sampling because the result of the second pick **is not** affected by the first pick).

- Sampling **without replacement** means that members of the population are not replaced after being selected and that each member cannot be chosen more than once. (known as **dependent** sampling because the result of the second pick **is** affected by the first pick).

- **Mutually exclusive** events cannot occur at the same time, meaning that they share no common outcomes and *P*(*A* and *B*) = 0.

Example 1. Events *A*, *B*, and *C* are all equally likely and mutually exclusive.

(a) Find *P*(*A*).

$\frac{1}{3}$

(b)Find *P*(A').

$= 1 - P(A)$
$= 1 - 1/3$
$= 2/3$

(c)Find *P*(*A* and *B*).

$\emptyset$

3.3: Two Basic Rules of Probability

- The **multiplication rule** - the probability of two events both happening, $P(A \text{ and } B)$, is equal to $P(A)P(B|A)$.

The Multiplication Rule for Independent Events:
If events A and B are independent, then $P(A \text{ and } B) = P(A) * P(B)$.

- The **addition rule** - the probability of either one of two events happening, $P(A \text{ or } B)$, is equal to $P(A) + P(B) - P(A \text{ and } B)$.

The Addition Rule for Mutually Exclusive Events:
If events A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.

Example 2. A spinner has 4 equal sectors colored yellow, blue, green, and red.

(a) What is the probability the spinner lands on yellow?

$P(Y) = \frac{1}{4}$

(b) What is the probability the spinner lands on red or blue?

$P(R \text{ or } B) = P(R) + P(B)$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

(c) What is the probability the spinner does NOT land on green?

$P(G') = 1 - P(G)$

$= 1 - \frac{1}{4} = \frac{3}{4}$